CS 762 Winter 2011 Wednesday March 9, 2011
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ASSIGNMENT 2

You will get more out of the problems if you try to do them yourself without looking them up anywhere. If you use any resources (books, papers, web sites, etc.) be sure to acknowledge them. And be sure to write the solution in your own words—the best way to be sure of this is to put any resources you may have used out of sight when you write your solution.

Please use separate sheets of paper for your solution to each part of each question.

1. Given a set of rectangles in the plane with horizontal and vertical sides, make a graph with a vertex for each rectangle, and an edge when two rectangles are intersected by a vertical line, or are intersected by a horizontal line.

   (a) Prove or disprove: these graphs are perfect.

   (b) Describe these graphs in relation to interval graphs.

   (c) [an open-ended bonus question, to which I do not know the answer] Can such graphs be recognized in polynomial time?

2. A graph $G = (V, E)$ is a *containment graph* if there is a family of sets $\mathcal{S} = \{S_v : v \in V\}$ such that $(u, v) \in E$ if and only if $S_u \subseteq S_v$. Prove that if G is a containment graph, then it is a containment graph of subtrees of a tree.

3. Let $G = (V, E)$ be a graph. A *dominating set* is a subset $D \subseteq V$ such that every vertex of $V - D$ is adjacent to some vertex of D. It is NP-complete to decide if a graph has a dominating set of size at most k.

   (a) Give a polynomial time algorithm to find a minimum dominating set in an interval graph.

   (b) Give a polynomial time algorithm to find a minimum dominating set in a permutation graph.