CS 762 Winter 2011 Wednesday March 30, 2011
A. Lubiw

ASSIGNMENT 3

You will get more out of the problems if you try to do them yourself without looking them up anywhere. If you use any resources (books, papers, web sites, etc.) be sure to acknowledge them. And be sure to write the solution in your own words—the best way to be sure of this is to put any resources you may have used out of sight when you write your solution.

Please use separate sheets of paper for your solution to each part of each question.

1. Consider the graphs obtained in the following way: Let T be a tree with at least 4 vertices, and with no vertex of degree 2. Take a planar embedding of T and add a cycle through all the leaves of the tree, following the natural cyclic order in the planar embedding.

   (a) Prove that these graphs have treewidth ≤ 3 by giving a direct construction of a tree-decomposition.

   (b) Show that no graph is this class has treewidth 2. [You might like to use the result that any minor of a graph of treewidth k also has treewidth k.]

2. Consider the graphs obtained by taking a planar cycle and adding non-crossing chords inside and outside the cycle (resulting in two outerplanar graphs sharing the same cycle).

   (a) Show that the graphs of question 1 are a special case of these graphs. [Hint: think about a Hamiltonian cycle.]

   (b) Show that these graphs do not have bounded treewidth. [You may use the fact that an $n \times n$ grid has treewidth n.]

3. Show that if a graph G has treewidth k, then it has a tree-decomposition where every tree node is contained in exactly $k + 1$ subtrees, and every tree edge is contained in exactly k subtrees. In other words, for every tree node u, its bag V_u has size $k + 1$, and for every tree edge (u, v), the intersection of the two bags, $V_u \cap V_v$, has size k. [You do not need to use the equivalence between treewidth k and partial k-trees—you can just give a direct proof by modifying the tree decomposition.]