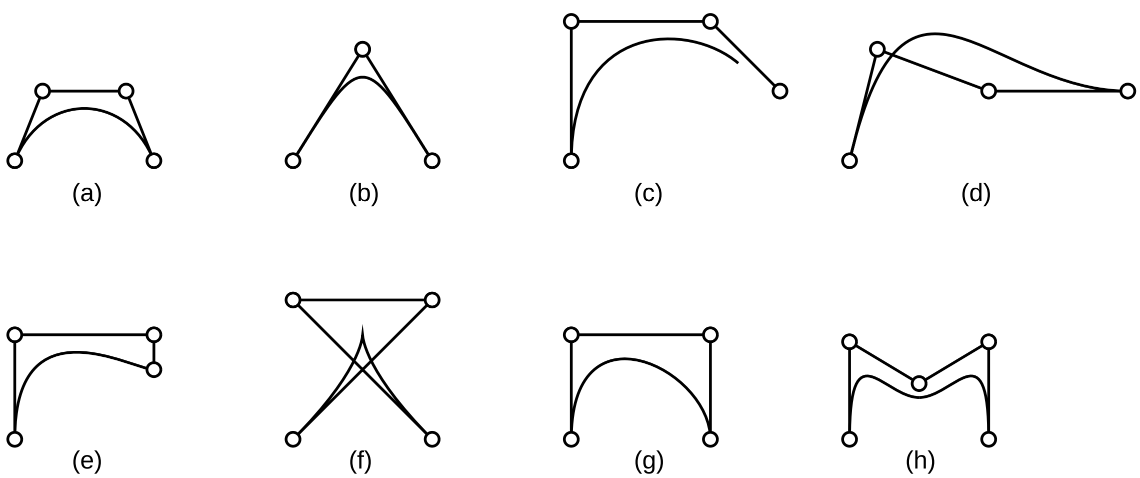
CS 779, Winter 2020
Assignment 2

1. (10 pts) Below are eight curves and their "control points." Some sets of control points are the Bézier control points for the corresponding curve. The others are not. Determine which are the Bézier points for the curve, and which are not. For those that are not Bézier control points for the curve, give a reason that they are NOT. Assume that no control point is duplicated.



2. (5 pts) In the Synthesis lecture, Figure 2.6 shows the de Casteljau evaluation of a polynomial, with the control points and intermediate points labeled with blossom values. This figure shows a complete evaluation of the polynomial $F(t)$.

   We can also use the blossom to evaluate off the diagonal. Given control points for a cubic polynomial F parameterized over $[0, 1]$,

$$\begin{aligned} P_0 &= (0,0) \\ P_1 &= (0,1) \\ P_2 &= (1,1) \\ P_3 &= (1,0), \end{aligned}$$

   draw a figure for the de Casteljau type of evaluation of f the blossom of F at $f(.25, .25, .5)$. Draw this three ways: once where the .5 is the parameter at the first level of evaluation, once where it is the parameter at the second level, and once where it is the parameter at the third level. Label all the intermediate points with their blossom values, and give the coordinates for all points of evaluation. As an alternative to giving the coordinates of each point of evaluation, you may draw the figures on graph paper (or a grid) with each square being of width 1/32.

3. (10 pts) Suppose we have a parametric cubic curve F in Bézier form and its triaffine blossom f. Let $G(u) = f(1-u, 1-u, u)$. G is a polynomial curve. Determine its degree and use blossoming to derive formulae for its Bézier control points over the interval $[0, 1]$. You should **not** expand either F or G relative to any basis; instead, you should try an approach similar to that used in the course notes for degree raising.

4. (10 pts) A planar parametric cubic Bézier curve may intersect itself. Prove or give a counter example to the following statement: A parametric cubic curve that intersects itself is planar.