CS 779, Winter 2020
Assignment 4

1. (5 pts) In the Synthesis Lecture, we considered the cubic Hermite interpolation problem when the knots were consecutive integers. Here we will generalize the problem to arbitrary knot vectors.

   Given points $P_0, \ldots, P_L$, vectors $\vec{v}_0, \ldots, \vec{v}_L$, and a knot vector $t_0 < \ldots < t_L$, find the cubic Bézier control points for the Cubic Hermite curve that interpolates this data. I.e., H is this Cubic Hermite curve if

   - $H(t_i) = P_i$
   - $H'(t_i) = \vec{v}_i$.

2. (5 pts) Give the B-spline control points and corresponding knot vector for the above curve. Note: it is inadequate to give the Bézier control points and the corresponding B-spline knot vector; you should remove any knots that can be removed without changing the curve.

3. (20 pts) Implement an interactive 2D B-spline editor using the Lane-Riesenfeld algorithm to evaluate the curve. Your program should have the following functionality:

   - The left mouse button adds a new control point.
   - The middle mouse button is used to move control points.
   - New segments have unit length parameterization (e.g., when adding a new control point, assume the value of any new knot to be one more than the last knot in the knot sequence).
   - There are two display modes:
     - Just the curve.
     - The curve and the control polygon.
   - There should be a reset key/menu-option that clears all the control points.
   - There should be two menus (or other UI) for controlling the Lane-Riesenfeld algorithm. The first menu should control the subdivision level; you should support at least subdivision levels 1 to 4.

     The second menu should control the degree of the B-spline. You should support at least quadratics, cubic, and quartic.

     Part of your UI should give feedback on what the current degree/subdivision level is.

You should submit a short write-up telling me where the executable is, how to run and operate it, and what functionality you implemented. You do not need to submit any code.

Optional Questions

The following question is optional. You might want to look at it, and think about them briefly. Do it and submit it if you want — I'll mark it if you do, and it won't hurt your grade. In fact, it might help your grade: if you do this question and miss points on the other questions on this assignment, I will use these to boost your score on this assignment up to 100%.

- (10 pts*) Prove that the arc length of a Bézier curve is always less than or equal to the perimeter of its control polygon. You may use the fact that when you repeatedly subdivide a Bézier curve at its mid-point, then in the limit, the control polygon(s) converge to the curve.