# CS 779, Winter 2020
# Assignment 5

1. (10 pts) The continuity theorems about B-splines tell us that if a knot $u$ has single multiplicity, then a B-spline of degree $n$ is $C^{<n-1>}$ at this point. However, this is clearly an analytic condition, not a geometric one, since if we make $n+1$ consecutive control points coincident, then we have introduced a segment of zero length (i.e., all derivatives are 0 and the curve is of zero length over a non-zero length interval in the domain) and we can introduce a cusp in the curve.

   - Suppose that we have the knot sequence $(0, 1, 2, 3, 4, 5, 6, 7)$ for a cubic B-spline $f$, and further suppose that $f(2, 3, 4) = f(3, 4, 5)$. The theorem on continuity tells us that this curve should be $C^2$ at $F(3)$. Is the curve geometrically $C^2$ at this point? If so, support your statement. If not, give a counter example.
   - Now suppose that $f(1, 2, 3) = f(2, 3, 4) = f(3, 4, 5)$. Is the curve geometrically $C^2$ at $F(3)$? If so, support your statement. If not, give a counter example.

2. (10 pts) Suppose you are given a cubic Bézier curve $F$ over the interval $[0, 1]$ and two points $P_0$, $P_1$, and you wish to extend $F$ to a 3-segment B-spline curve $N$ with breakpoints $\{-1, 0, 1, 2\}$ where

   - The first B-spline control point of the extended curve is $P_0$ and the last B-spline control point of the extended curve is $P_1$.
   - $N(-1) = P_0$, $N(2) = P_1$;
   - $N(t) = F(t)$ for $t \in [0, 1]$;
   - $N$ is a $C^2$ curve.

   (a) (2 pts) Give the knot vector for $N$.

   (b) (4 pts) Draw a diagram of a sample cubic, showing the control points of $F$, the construction of and the control points of $N$, and give a brief verbal description of how to construct the control points of $N$.

   (c) (4 pts) Give one (or two) triangle diagrams showing how to efficiently compute the control points of $N$. Be sure to circle the control points of $N$ in your digram.

3. (30 pts) Extend your interactive B-spline editor of the previous assignment in the following ways:

   - Implement B-splines for arbitrary knot vectors, using either de Boor's algorithm or evaluation of the basis functions.
   - The right mouse button displays the blossom value of the displayed control point closest to the current mouse position.

- There are three display modes:
  - Just the curve.
  - The curve and the control polygon.
  - The curve, the control polygon, and the control polygons for the corresponding Bézier curves.
- Somewhere, you should display the knot vector on a line. You should be able to use the middle mouse button to move existing knots of the knot vector, the left mouse button to add a new knot, and the right mouse button to display the value of the closest knot.
- There should be a toggle for displaying the blossom arguments of all displayed control points (including the Bézier control points, if they are currently being displayed). Note that if all blossom arguments are being displayed, then the right mouse button does nothing.
- There should be a 'reset' key/menu-option that clears all knots and control points.

You should submit a short write-up telling me where the executable is, how to run and operate it, and what functionality you implemented. You do not need to submit any code.

You may not use the OpenGL command `glunurbscurve` to draw the curve; you should sample the curve and draw it as a sequence of line segments.

You only need to draw cubic B-splines.