CS 779, Winter 2020
Assignment 9

1. (20 pts) Implement one of the following surface construction schemes:

   - Shirman-Séquin
   - Nielson
   - Triangular Gregory Patches

   Note while Shirman-Séquin is probably the hardest to implement, you can use the tesselator you wrote for the previous problem to render the surface. For the other two schemes, you'll need to integrate tesselation code into the scheme itself.

   To compute surface normals, you'll have to write numerical approximation code for all but Shirman-Séquin's code.

   You should fit your surfaces to the corners of the patches given in Problem 2 of assignment 8.

2. (35 pts) Suppose we want to construct a function $f(x, y)$ with the following properties:

   - f is zero on the boundaries and exterior of a regular n-sided polygon centered at the origin in the x-y plane.
   - f is non-zero everywhere inside the polygon.
   - $f(0, 0) = 1$.
   - f is C^1 everywhere.

   Further suppose we wish to construct such a function using quadratic (degree 2) triangular Bézier patches. As a first step we need to triangulate the domain polygon. We will then place one Bézier patch over each triangle.

   (a) (5 pts) Prove that if we use the obvious triangulation of the domain (connecting the center to the corners) then we can not construct quadratic Bézier patches over these triangles that meet C^1. Note: you only need to prove this for a particular polygon (I suggest the square). The generalization to an arbitrary n-gon is immediate.

   (b) (5 pts) Prove that we can not construct cubic Bézier patches over the triangles of the obvious triangulaion that meet C^1.

   (c) (5 pts) Derive a construction for such a function f using quartic (degree 4) triangular Bézier patches over the obvious triangulation.

   (d) (5 pts) Write a program to create the triangular Bézier patches in s3d format for the quartic solutions for $n = 3, 4, 5, 6$.

(e) (10 pts*) Derive a construction for such a function f using quadratic (degree 2) triangular Bézier patches. You may orient the polygon in the plane in any fashion that is convenient to you. Note that this question can be solved by using Powell-Sabin's scheme; you, however, are to find a solution that uses no more than $5n$ (although the problem can be solved with $3n$ patches). Note any shape parameters your construction has.

(f) (5 pts) Write a program to create the triangular Bézier patches in s3d format for the quadratic solutions for $n = 3, 4, 5, 6$.

Note: This problem uses functional Bézier patches. Once you know the domain of such a patch, you know the x-y values of all the control points. All you need to find is the z-values of the control points.

For the programming parts, if you write your solutions to a file as Bézier patches in s3d format, then you can use cglv to display the results directly. Further, you can use cglv to display the control points of the patches. There is no need to triangulate your resulting patches.