B-Spline Approximations

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CS779 Final Project

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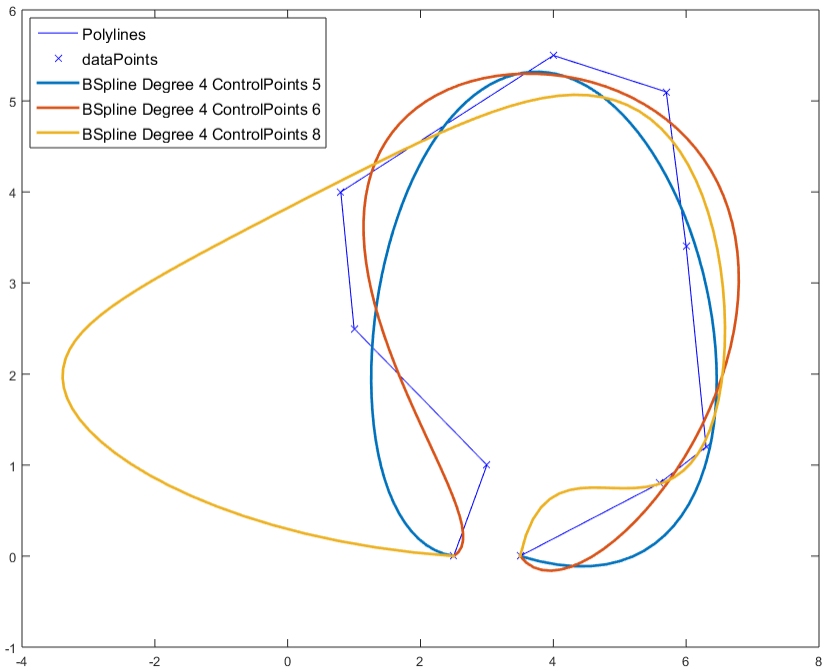
Problem Statement

Given data points $D_0, \cdots, D_m$, find the "best" B-spline curve to approximate the data



Concerns:

1. Degree
2. Number of control points p
3. Knots $t_0, \cdots, t_{p+1}$
4. Parameterization $u_0, \cdots, u_m$

Formulating an Optimization Problem

Given:

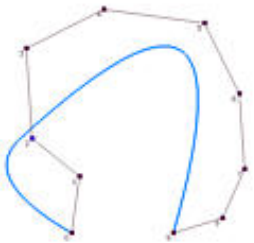
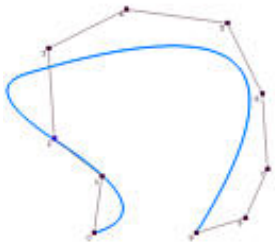
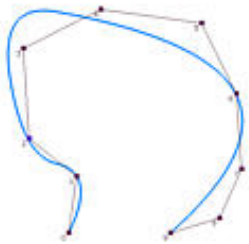
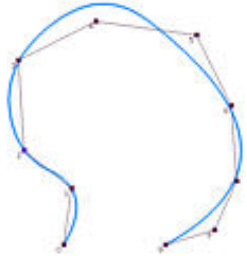
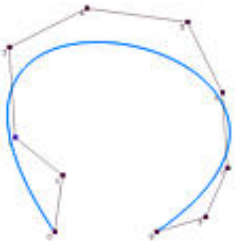
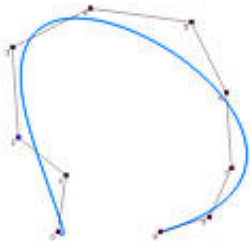
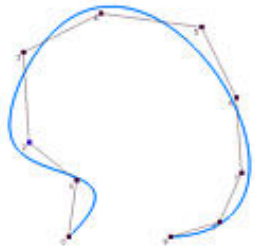
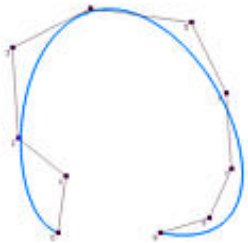
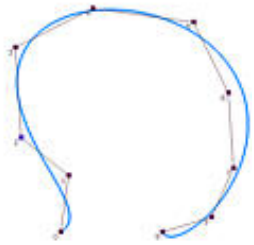
1. n - degree
2. p - number of control points
3. $t_0, \cdots, t_{p+n-1}$ - knot vector
4. $D_0, \cdots, D_m$ - data points
5. $u_0, \cdots, u_m$ - parameterization

Can find least-squares approximation:

$$\min f(P_0, \cdots, P_{p-1}) = \sum_{k=1}^{m-1} |D_k - C(u_k)|^2$$

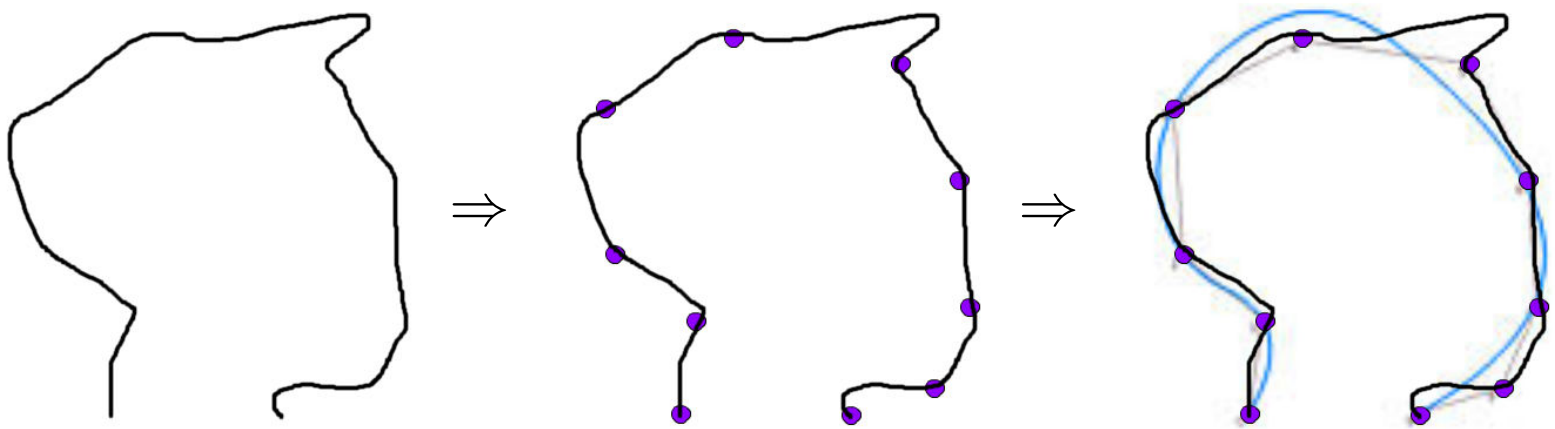
Optimal solution found by solving a system of linear equations.

Degree and Control Points

	4 CPs	5 CPs	6 CPs	7 CPs
degree 2				
degree 3				
degree 4				
degree 5				

My Project

1. User inputs curve
2. Sample points on curve
3. Compute low degree B-spline approximation with few control points



Bonus Features

- Degree and number of control points selected by user-defined preferences, change objective function to

$$\min f(P_0, \cdots, P_{p-1}) + g(p) + h(n)$$

- Detect closed curves
- Allow cusps, corners
- Incorporate a set of "known" curves into the objective function
- Different objective functions than sum of squared distance

Learning Opportunities

- B-spline approximation schemes
- Whether approximation "looks" much better than interpolation
- Effect of knots, parameterization on shape of curve
- Convex optimization

Thanks for listening.