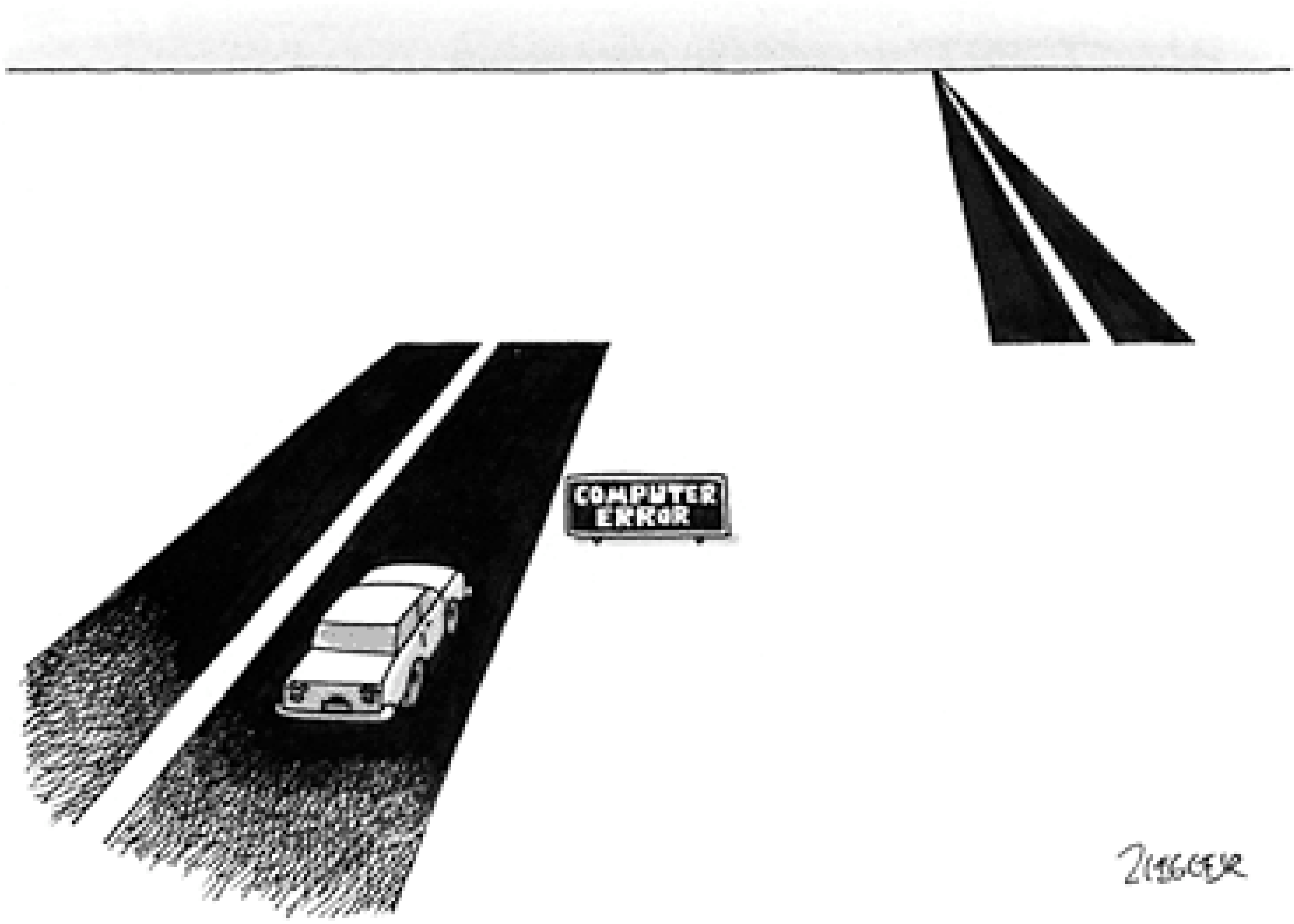
Error in Calculations



Today's Lecture

1. Intro to Software Engineering
2. Inexact quantities
3. Error propagation
4. Floating-point numbers
5. Design process
6. Teamwork
7. Project planning
8. Decision making
9. Professional Engineering
10. Software quality
11. Software safety
12. Intellectual property

Agenda

- Inexact measurements
- Writing inexact quantities
- Computing with inexact quantities

Readings

IPE Ch. 10 - Measurements and Units

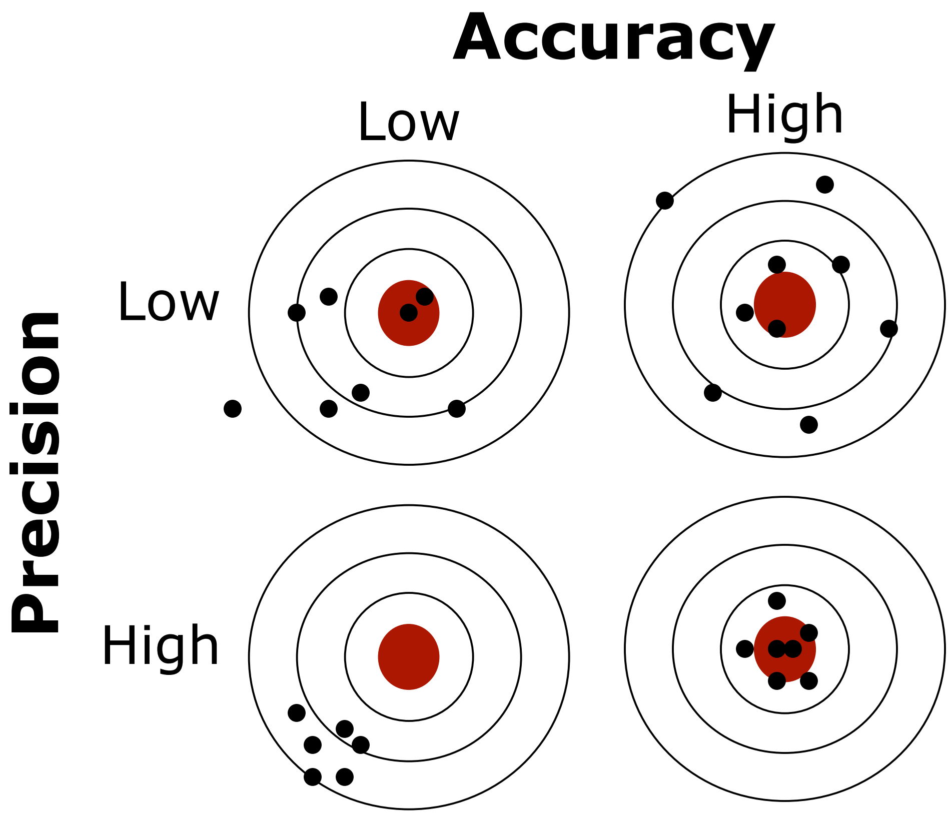
IPE Ch. 11 - Measurement Error

Error (Uncertainty)

The word **error**, with respect to numerical quantities, does not have the usual connotations of **mistake.**

Rather, **error** in scientific measurement refers to the degree of **uncertainty,** or **inexactness,** in a measured value.

Measurement Uncertainty



These two words are often loosely used as synonyms, but beware of using the term *precision* when you mean *accuracy*.

Andrews, Aplevich, Fraser, and Ratz, *Introduction to Professional Engineering*, Faculty of Engineering, 1997.

Importance of Knowing Uncertainty

Engineers design products, make decisions, and give advice based on **inexact measured quantities**.

Example: Determine whether a crown is made of 18-karat gold, as claimed, or made of a cheaper alloy.

Facts:

$\rho_{gold} = 15.5$ gm/cm^3

$\rho_{alloy} = 13.8$ gm/cm^3

Measurements reported:

Expert A: $\rho_{crown} = 15 \pm 1.5$ gm/cm^3

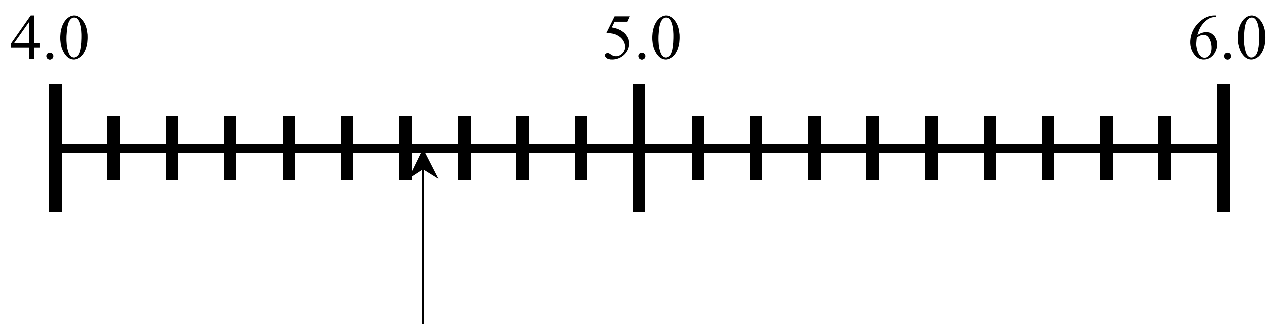
Expert B: $\rho_{crown} = 13.9 \pm 0.2$ gm/cm^3

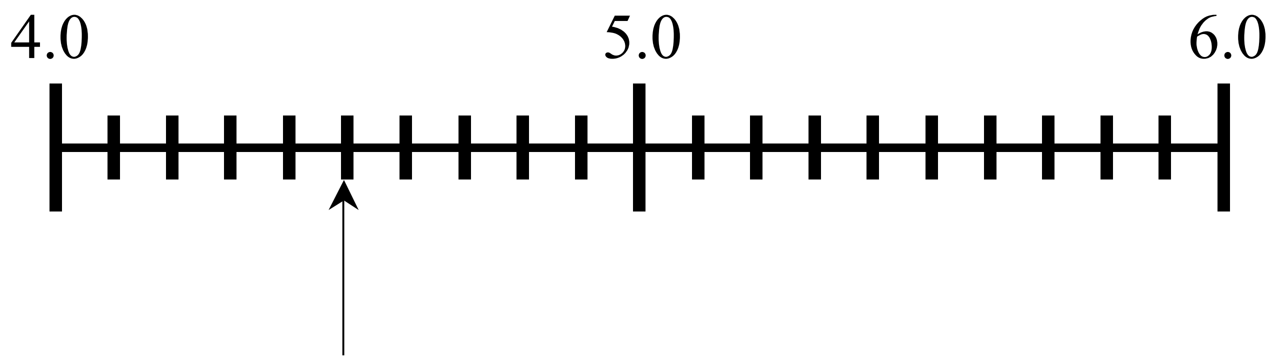
Inevitability of Uncertainty

It is impossible to measure any attribute **exactly,** (i.e., without uncertainty).

- Limited by the precision of our measurement tools and techniques

Inevitability of Uncertainty





Inevitability of Uncertainty

It is impossible to measure any attribute **exactly,** (i.e., without uncertainty).

- Limited by the precision of our measurement tools and techniques

- Measurement may be affected by external factors (temperature, humidity, air pressure, motion)

- Attribute being measured may not be uniform (e.g., the density of the crown may be different in different places).

Reporting Uncertainties

By direct statement

$$v_i = 112.4 \pm 0.2 \text{ V}$$

indicates an uncertainty of 0.2 V

By indirect statement

$$v_i = 112.4 \text{ V}$$

implies an error of **±1/2 of the least significant digit:**

$$v_i = 112.4 \pm 0.05 \text{ V}$$

Reporting Uncertainties

Scientific and engineering notations were introduced to **reduce the ambiguity** of error in reported quantities.

Scientific Notation

v_i = 1.124 x 10^2 V v_i = 1.1240 x 10^2 V

- One nonzero digit to the left of the decimal
- The power of 10 is a scale factor
- The number of digits is exactly the number of significant digits

Engineering Notation

v_i = 112.4 x 10^0 V v_i = 0.1124 x 10^3 V

- The exponent is a multiple of 3, corresponding to the SI prefixes

Significant Digits (in Uncertainty)

Absurd: 9.82 ± 0.02385 m/sec^2

Rule: Experimental uncertainty should usually be rounded to **one** significant digit.

Unless the leading digit in the uncertainty is small (1 or 2), in which case the uncertainty may be rounded to **two** significant digits:

5.1 ± 1.4 m/sec

John R. Taylor, *An Introduction to Error Analysis*, 1997

Significant Digits (in Measurement)

Absurd: 3869.71 ± 30 m/sec

Rule: The least significant digit in the measured value should be of the same order of magnitude as the uncertainty.

Unless the leading digit in the uncertainty is small (1 or 2), in which case it may be appropriate to retain one extra digit in final answer:

12.6 ± 1 m/sec

John R. Taylor, *An Introduction to Error Analysis*, 1997

Estimating Uncertainties

Repeatable measurements

2.3, 2.4, 2.5, 2.4

The mean of N measured quantities is a more accurate estimate than a single sample.

Rule: The uncertainty in the mean of N measurements is $1/\sqrt{N}$ of the uncertainty in one measurement.

Andrews, Aplevich, Fraser, and Ratz, *Introduction to Professional Engineering*, Faculty of Engineering, 1997.

Example: Lego Light-Sensor Readings

Problem: Determine the light-sensor readings that correspond to ground colours blue and black.

Experimental readings (blue):
42, 39, 45, 40, 42, 44, 43, 40, 42, 43

Experimental readings (black):
42, 44, 46, 45, 44, 47, 42, 44, 45, 41

Problem: Determine whether the light sensor is sensing colour blue or black.

Experimental reading: 42

Propagating Uncertainties (Addition)

Rule: The sum of two measured values is

$$(x \pm \Delta x) + (y \pm \Delta y) = (x+y) \pm (\Delta x + \Delta y)$$

Propagating Uncertainties (Subtraction)

Rule: The difference of two measured values is

$$(x \pm \Delta x) - (y \pm \Delta y) = (x - y) \pm (\Delta x + \Delta y)$$

Propagating Uncertainties (Product)

Rule: The product of two measured values is

$$(x \pm \Delta x)(y \pm \Delta y) = (xy)(1 \pm [\Delta x/|x| + \Delta y/|y|])$$

Propagating Uncertainties (Division)

Rule: The quotient of two measured values is

$$(x \pm \Delta x)/(y \pm \Delta y) = (x/y)(1 \pm [\Delta x/|x| + \Delta y/|y|])$$

Propagating Uncertainties (Times k)

Rule: The product of a measured value and a constant *k* is

$$k(x \pm \Delta x) = kx \pm |k|\Delta x$$

Propagating Uncertainties (Powers)

Rule: The power of a measured value is

$$(x \pm \Delta x)^n = x^n \left(1 \pm n \left(\Delta x / |x|\right)\right)$$

Example: QUEST

SISP, the UW student information system, records students' marks and averages.

Marks:

91
89
79
86
92

Guard Digits in Calculations

Extra digits can be carried through intermediate calculations, either

- one or two guard digits, or
- the full capacity of the hardware arithmetic

then the final answer is rounded to the appropriate number of digits.

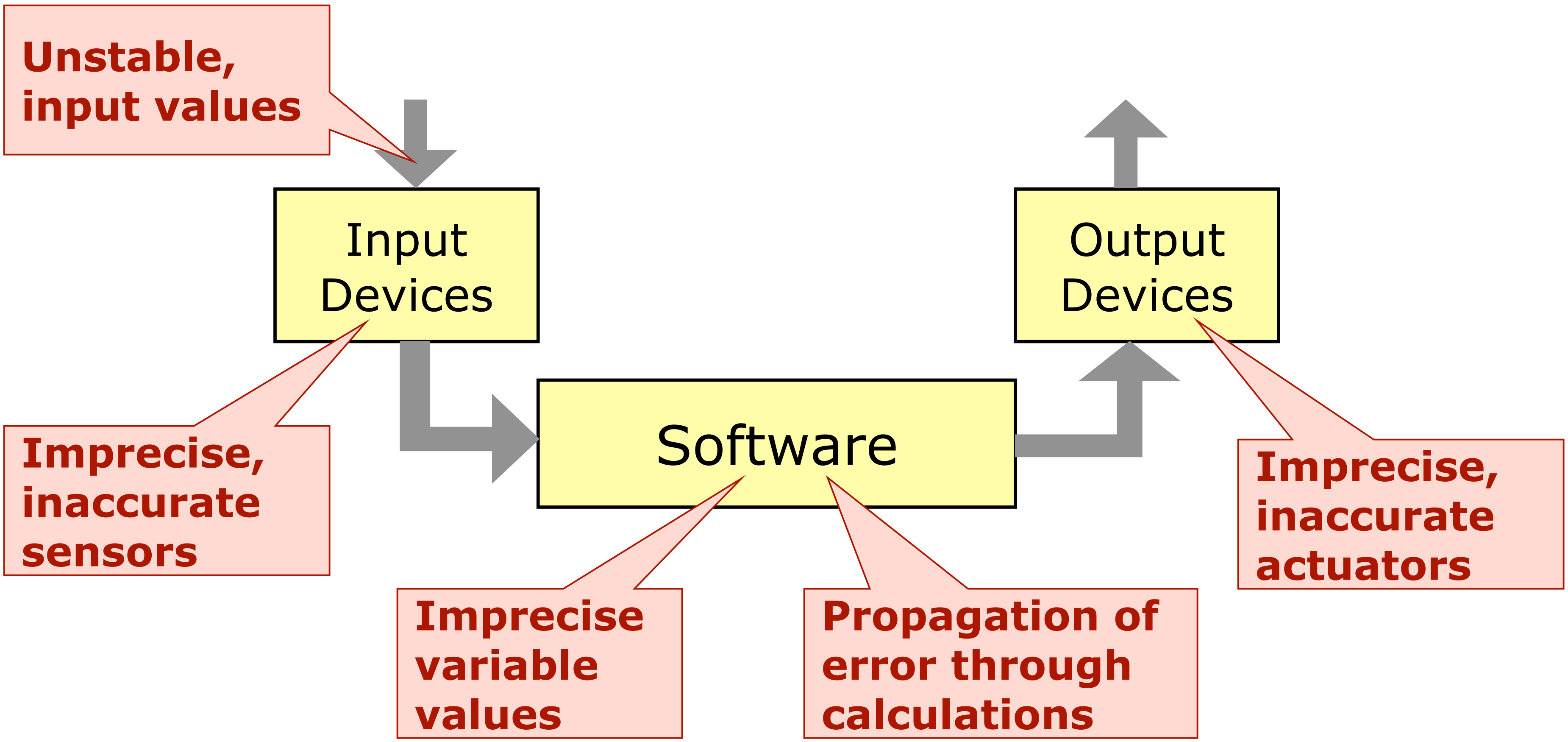
Rounding

Rule: Rounding drops insignificant digits, but may increase the least significant digit by 1:

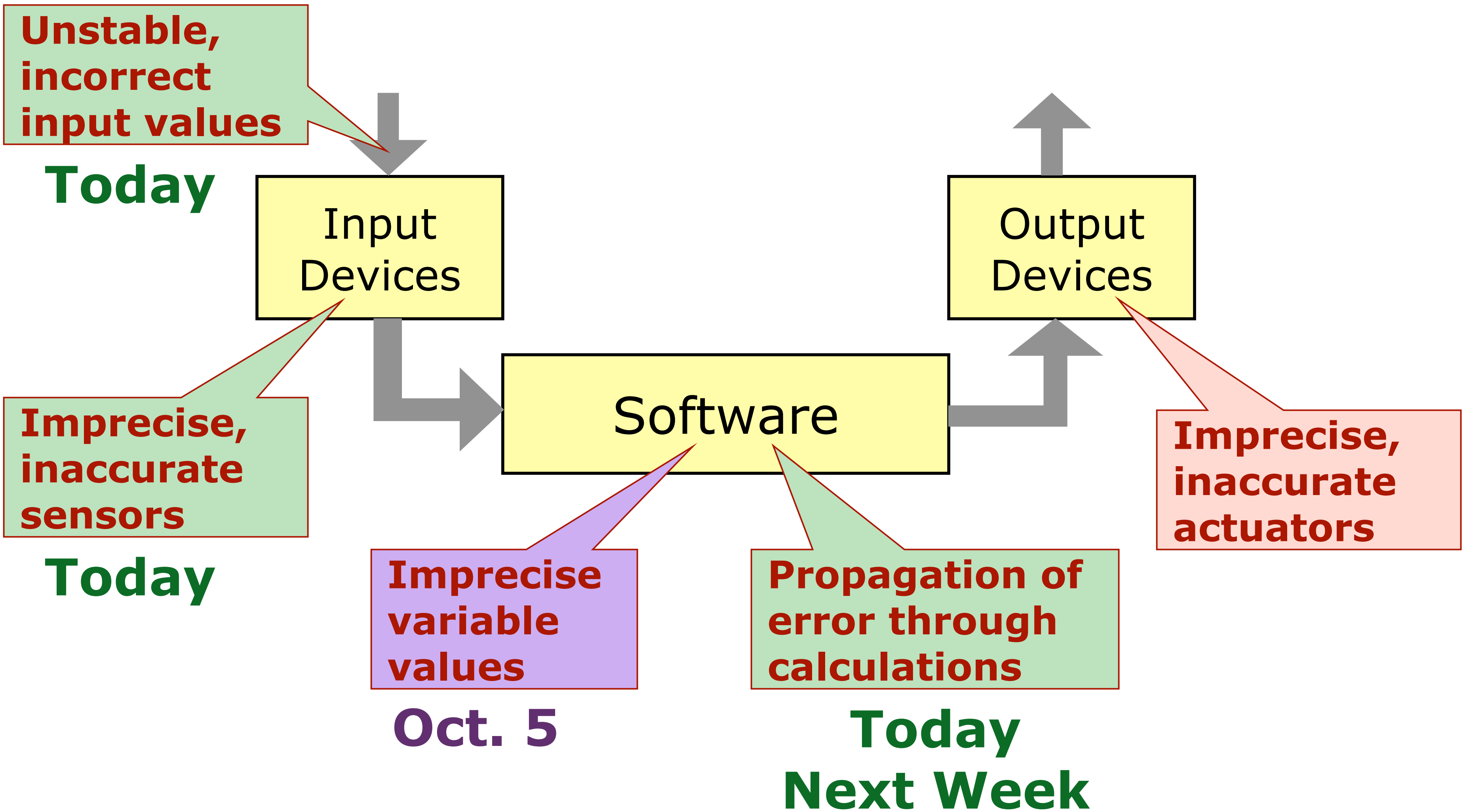
$$\text{Round}(d_1 d_2 d_3 d_4) = \begin{cases} d_1 d_2 & \text{if } d_3 d_4 < 50 \\ \mathbf{d_1 d_2} & \textbf{if } \mathbf{d_3 d_4 = 50} \textbf{ and } \mathbf{d_2} \textbf{ is even} \\ \mathbf{d_1(d_2+1)} & \textbf{if } \mathbf{d_3 d_4 = 50} \textbf{ and } \mathbf{d_2} \textbf{ is odd} \\ d_1(d_2+1) & \text{if } d_3 d_4 > 50 \end{cases}$$

This rule tends to preserve the mean value of a set of rounded numbers better than rounding up when $d_3 = 5$.

Sources of Uncertainty in Software



Sources of Uncertainty in Software



Summary

Simple rules for estimating uncertainties, calculating with uncertainties

Next week, we will learn more sophisticated techniques for calculating uncertainties that result in more precise uncertainties.

Readings

Next week's web review

Measurements and Measurement Error

IPE: Ch. 10, 11

Sentence structure

Dupré 1, 7, 8, 79, 85, 97

Next week's lecture

Error propagation

IPE: Ch. 12